Counting and Computing

Keith Fligg

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Part 1: Counting

- Review of Numbers

Part 2: Computing

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- Numerical Representation in the Brain

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- What is an Algorithm?
- What Problems are Computable (solvable)?

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- $\mathbb{N}$ – Natural Numbers
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- $\mathbb{Z}$ – Integers
  The integers are $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ and are also infinite and countable.
- $\mathbb{R}$ – Real Numbers
  The reals are all infinite decimal representations of numbers on the interval $(-\infty, +\infty)$. They are infinite and uncountable.

Numerical Representation in the Brain

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- Large, close numbers take longer to order.
- Abstract representations can be ordered.

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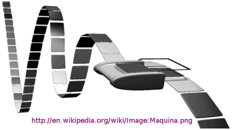
Alan Turing (1912-1954): Mathematician

www.alanturing.net

- Turing Machine



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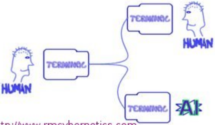

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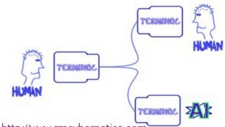

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  Dylan: I'm a grinder soldier.
  Me: What does a grinder soldier do?
  Dylan: That means I eat crime.
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  Dylan: Oranges.

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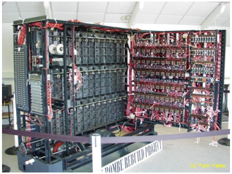

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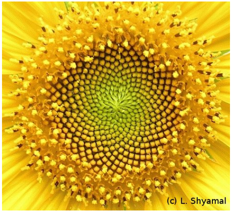

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- Examples: FOIL, TSP, others?

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- In 1970 Yuri Matijasevič proved that there is no algorithm to determine if a polynomial has integral roots.
- So, how do animals without computers solve problems (compute algorithms)? And how do they manage to solve all of their problems when we know that many of their problems can't be computed?